

IE 24 Lecture Notes
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General Topic: Location decisions

General Examples:

- Locate stores where they can capture the marketplace and generate optimal amount of sales.
- Locate places close to where the products are needed most.

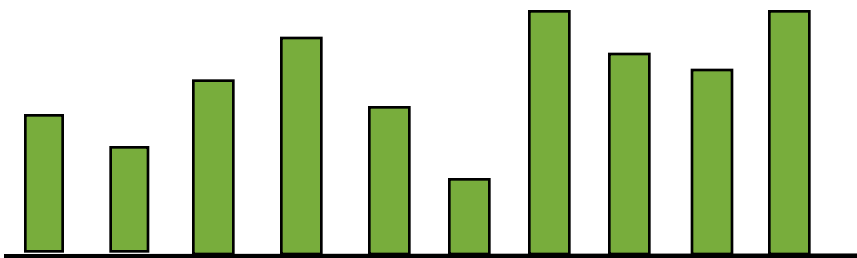
IEOR engineers have a lot of interest in location decisions and many industrial engineers research about how to make the best decision.

Class quiz: Choose the appropriate objective for each client.

- Objectives: 1. Minimize total travel distance of customers.
2. Minimize the maximum distance from the client to customer.
3. Maximum coverage area.
- Clients: 1. Verizon, 2. Hospital/service provider 3. Fire station
- Decision: 1. Verizon chooses the objective of maximum coverage area.
2. Hospital/service provider chooses to minimize the maximum distance traveled by customer.
3. Fire station chooses to minimize total travel distance of customers.

Coverage area is the area where services can be provided and the area is predetermined. Also, greater coverage area can be better depending on the circumstances like cost of expanding coverage area relative to increase in profits due to the area. Examples of coverage area in media include the advertisement comparing of Verizon 3g coverage of 80% of United States with that of AT&T which is 10%. The goal of any firm is to maximize coverage area while trying to minimize cost. The objective in the fire station's case is to minimize maximum distance so that that far off neighborhood don't burn down.

How to find the optimal location for a business based on graph of the linear density of people in one area based on location?



The green bars indicate the number people at each block(position) and we are supposed to find the optimal location that reaches that is easiest for the most number of customers to reach.

There are many ways to find the optimal locations but the proposed ideas in class were:

1. Use a approach similar to center of mass like in physics to find the location.
2. Use mathematical model to find the location. (This method is not practical for a simple problem like above due to the work required to create a model).

After deciding to use the 1st way, the professor defined some terms to create a formula for the total distance traveled by all the customers:

W_i = number of people in each block.

X_s = location of our business.

X_i = location of our customer

$$\sum_{i=1}^n W_i |X_s - X_i| = \sum_{i=1}^n W_i (X_s - X_i) + \sum_{i=s+1}^n W_i (X_i - X_s)$$

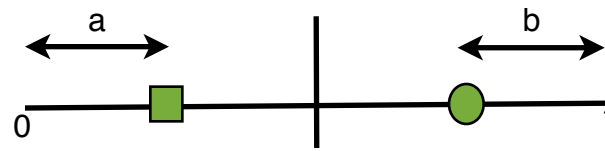
$|X_s - X_i|$ is the distance traveled by each customer.

W_i is the population at each block.

To find the optimal location of the business (X_s), take the derivative of the above function then set it equal to 0. The next step is to solve for X_s .

Let's take a more complex example with a similar population distribution as the above illustration. In this case, we are assuming that there is a original beer stand on a beach and a new vendor comes in sell beer in the same general area. Taking into account that both vendors sell the same beer at the same price, Where should both of the vendors locate the beer stands? The new comer should locate his beer stand closer to a dense population. According to human psychology, each vendor tries to move closer to the other to lure more customers away from the other. In the end, both vendors end up in the same location.

In a new scenario with two vendors, the population is uniformly distributed in a given area. So, the illustration in the last page will have bars of same length. Where should the beer vendors locate their stands to have share the customers evenly? There are infinitely many solutions as long as both vendors are located the same distance away from the midpoint. Human behavior causes the each vendor move closer to other vendor to lure customers from him. Eventually, both vendors keep moving until both of them end up in the middle. The research into how humans make economic decision is called behavioral economics. This field researches how humans make decisions based on other factors besides logic.



Customers going to the square vendor = $a + (1-b-a)/2 = (a - b + 1)/2$

Customers going to the circle vendor = $b + (1-b-a)/2 = (1+b-a)/2$

Setting both equations equal to each other, you get:

$$(a - b + 1)/2 = (1 + b - a)/2$$

After solving, you get $a = b$.

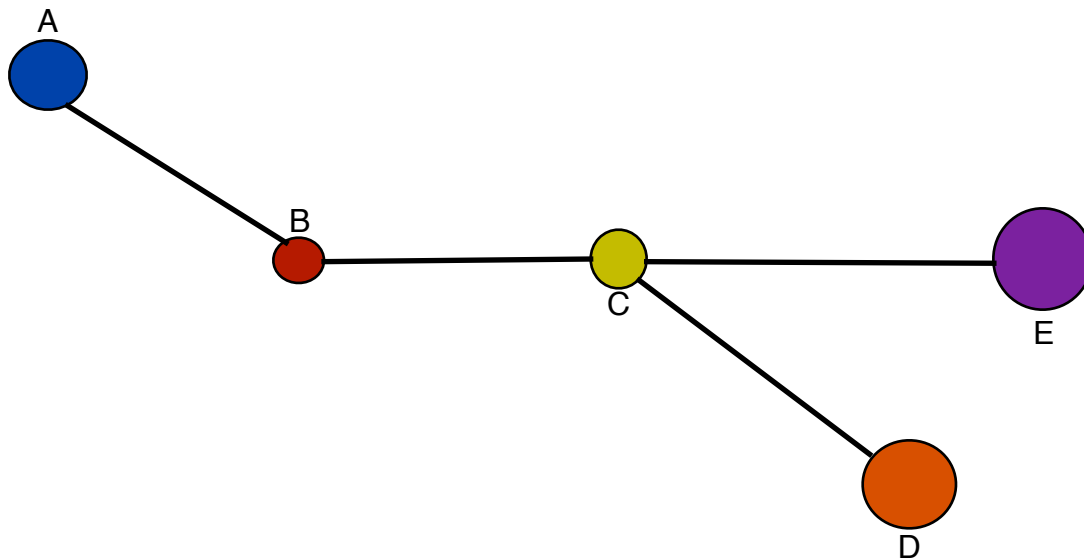
So, the optimal location is when both vendors are the same distance away from the endpoints (or center). Both vendors will move closer and closer as human behavior suggests and both of them end up in the middle. Example of this would be gas stations being in the same general location. Being in the same location is also an optimal solution since both are the same distance from the center.

Recent research:

- The development of location models for IBM plants in China and warehouses. The company spends hundreds of millions of dollars to build a plant and it wants to locate it in the best possible location. A wrong decision made will cause the company to lose millions and it will cost many more millions to relocate the plant.
- Professor Shen worked with multiple teams of other professors and consultants to find the best location for a new pharmaceutical plant that costs \$ 500 million.

Example: Airlines using location models to find the city where to locate their hubs.

The Airlines use a hub&spoke network where a passenger travels from a small city to a hub then on to another small city. It is not feasible to have to have direct flights between two small cities. For example, you cannot have direct flights between 200 cities since it would require the airline to have 40,000 flights. So, the airlines group all the passengers from different cities in their hub and then send them together to the desired small city so that the planes are full.



Where should an airline locate their hub considering the five cities above represented by circles?

Data:

- Population: A = 10, B = 5, C = 7, D = 12, & E = 14.
- Distance: AB = 8, BC = 7, CD = 5, & CE = 10.

The best location for a hub will be city C because it is close to the big population centers and it is not too far away from the smaller cities.

In more complex real life situations, there are too many nodes to keep track of them in our minds. It is extremely hard to pick the optimal solution just based on simple logic and trial&error. Intuition is also needed in addition to computers.

What kind of algorithm can you create to handle thousands of nodes?

-You can use the same method used in the beer stand example but in 2D. You can use center of mass then find the actual point closest to the center of mass.

- Simple solution: 1. Pick a tip node (doesn't have to 2 or more connections)
- 2. If the population at the tip node is greater than the total population divided by 2, the node is where we will place our hub.
- 3. If the above condition is not true, eliminate the node as a possible choices.
- 4. Choose the next tip node and its population will be the population of the eliminated node plus its own population. Repeat the process to find the optimal location.